Internet Appendix for
"Countercyclical Income Risk and Portfolio Choices: Evidence from Sweden"

SYLVAIN CATHERINE, PAOLO SODINI, and YAPEI ZHANG*

Abstract

This Internet Appendix contains one section which provides additional tables and figures supporting the main text.

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This section derives the relationship between co-moments of the human capital shock and stock market return distribution, and the market beta of human capital. We assume an agent with CRRA γ, wealth W, and labor income Y_t. For simplicity, we study the case of an agent investing only in the risk-free asset.

We denote the certainty equivalent at time $s < t$ of labor income from time t onward by $H_{s,t}$. For parsimony, we use the notation H_t for $H_{t,t}$. The expected utility of the agent at time t is of the form

$$V_t = a_t \frac{(W_t + H_t)^{1-\gamma}}{1-\gamma}, \tag{IA1}$$

where a_t is a function of the agent's age.

In the previous period, the Bellman equation of the agent is

$$V_{t-1} = u_{t-1} + \beta \mathbb{E}_{t-1}[V_t], \tag{IA2}$$

where β is the agent's discount factor. By definition of $H_{t-1,t}$, we have

$$V(W_t + H_{t-1,t}) = \mathbb{E}_{t-1}\left[V(W_t + H_t)\right], \tag{IA3}$$

which is equivalent to

$$\frac{(W_t + H_{t-1,t})^{1-\gamma}}{1-\gamma} = \mathbb{E}_{t-1}\left[\frac{(W_t + H_t)^{1-\gamma}}{1-\gamma}\right]. \tag{IA4}$$

We denote the expected value of H_t as of period $t-1$ by $\overline{H}_t$. We apply a first-order Taylor expansion on the left-hand side and third-order expansion on the right-hand side around $(W_t + \overline{H}_t)$:

$$
\begin{gathered}
U(W_t+\overline{H}_t)+U'(W_t+\overline{H}_t)(H_{t-1,t}-\overline{H}_t) \\
= \\
\mathbb{E}_{t-1}\Big[U(W_t+\overline{H}_t)+U'(W_t+\overline{H}_t)(H_t-\overline{H}_t)+ \\
\frac{1}{2}U''(W_t+\overline{H}_t)(H_t-\overline{H}_t)^2+\frac{1}{6}U'''(W_t+\overline{H}_t)(H_t-\overline{H}_t)^3\Big].
\end{gathered}
\tag{IA5}
$$

Applying the linearity of expectations and rearranging equation (IA5):

$$
\begin{gathered}
V'(W_t+\overline{H}_t)(H_{t-1,t}-\overline{H}_t) \\
= \\
V'(W_t+\overline{H}_t)(H_t-\mathbb{E}_{t-1}[\overline{H}_t])+\frac{1}{2}V''(W_t+\overline{H}_t)\mathbb{E}_{t-1}\Big[(H_t-\overline{H}_t)^2\Big] \\
+\frac{1}{6}V'''(W_t+\overline{H}_t)\mathbb{E}_{t-1}\Big[(H_t-\overline{H}_t)^3\Big],
\end{gathered}
\tag{IA6}
$$

Rewriting the squared and cubic term as moments, we obtain

$$
\begin{aligned}
V'(W_t+\overline{H}_t)(H_{t-1,t}-\overline{H}_t) &= V'(W_t+\overline{H}_t)(H_t-\overline{H}_t) \\
&\quad +\frac{1}{2}V''(W_t+\overline{H}_t)\mathrm{Var}_{t-1}(H_t)+\frac{1}{6}V'''(W_t+\overline{H}_t)\mathrm{Skew}_{t-1}(H_t).
\end{aligned}
\tag{IA7}
$$

Rearranging and dividing both sides by $V'(W_t+\overline{H}_t)$, we have

$$
H_{t-1,t}=\overline{H}_t+\frac{1}{2}\frac{V''(W_t+\overline{H}_t)}{V'(W_t+\overline{H}_t)}\cdot\mathrm{Var}_{t-1}(H_t)+\frac{1}{6}\frac{V''(W_t+\overline{H}_t)}{V'(W_t+\overline{H}_t)}\cdot\mathrm{Skew}_{t-1}(H_t).
\tag{IA8}
$$

Given our CRRA utility function, the equation above can be simplified as

$$H_{t-1,t} = \overline{H}_t - \frac{\gamma}{2\left(W_t + \overline{H}_t\right)} \cdot \mathrm{Var}_{t-1}(H_t) + \frac{\gamma(\gamma+1)}{6\left(W_t + \overline{H}_t\right)^2} \cdot \mathrm{Skew}_{t-1}(H_t), \quad \text{(IA9)}$$

where Skew refers to the third central moment, or unstandardized skewness.

Now, consider some news in period $t-1$ that affects both the stock market and the agent's perception of the distribution of H_t. The effect on the certainty equivalent is

$$\Delta H_{t-1,t} = \Delta\overline{H}_t - \frac{\gamma}{2\left(W_t + \overline{H}_t\right)} \cdot \Delta\mathrm{Var}_{t-1}(H_t) + \frac{\gamma(\gamma+1)}{6\left(W_t + \overline{H}_t\right)^2} \cdot \Delta\mathrm{Skew}_{t-1}(H_t), \quad \text{(IA10)}$$

and the immediate return is

$$\frac{\Delta H_{t-1,t}}{H_{t-1,t}} = \frac{\Delta\overline{H}_t}{H_{t-1,t}} - \frac{\gamma}{2}\omega_H \cdot \Delta\mathrm{Var}_{t-1}(\epsilon_t) + \frac{\gamma(\gamma+1)}{6}\omega_H^2 \cdot \Delta\mathrm{Skew}_{t-1}(\epsilon_t), \quad \text{(IA11)}$$

where $\omega_H = \frac{\overline{H}_t}{W_t + \overline{H}_t}$ is the share of human capital in the agents total wealth and $\epsilon_t = \frac{H_t}{\overline{H}_t}$ is the scaled stochastic shock to the value of human capital at the beginning of period t. From this, given the definition of Covariance, we can use the following expression

$$\mathrm{Cov}\left(\frac{\Delta H_{t-1,t}}{H_{t-1,t}}, r_s\right) = \mathbb{E}\left[\frac{\Delta H_{t-1,t}}{H_{t-1,t}} \cdot r_s\right] - \mathbb{E}\left[\frac{\Delta H_{t-1,t}}{H_{t-1,t}}\right] \cdot \mathbb{E}[r_s], \quad \text{(IA12)}$$

which is equivalent to

$$
\begin{aligned}
\mathrm{Cov}\left(\frac{\mathit{\Delta} H_{t-1,t}}{H_{t-1,t}}, r_s\right) &= \mathbb{E}\left[\frac{\mathit{\Delta}\overline{H}_t}{H_{t-1,t}} \cdot r_s\right] - \frac{\gamma}{2}\omega_H \cdot \mathbb{E}\left[\mathrm{Var}_{t-1}(\epsilon_t) \cdot r_s\right] \\
&+ \frac{\gamma(\gamma+1)}{6}\omega_H^2 \cdot \mathbb{E}\left[\mathit{\Delta}\mathrm{Skew}_{t-1}(\epsilon_t) \cdot r_s\right] \\
&- \mathbb{E}\left[\frac{\mathit{\Delta}\overline{H}_t}{H_{t-1,t}}\right] \cdot \mathbb{E}[r_s] + \frac{\gamma}{2}\omega_H \cdot \mathbb{E}\left[\mathrm{Var}_{t-1}(\epsilon_t)\right] \cdot \mathbb{E}[r_s] \\
&- \frac{\gamma(\gamma+1)}{6}\omega_H^2 \cdot \mathbb{E}\left[\mathit{\Delta}\mathrm{Skew}_{t-1}(\epsilon_t)\right] \cdot \mathbb{E}[r_s] \\
&= \left(\mathbb{E}\left[\frac{\mathit{\Delta}\overline{H}_t}{H_{t-1,t}} \cdot r_s\right] - \mathbb{E}\left[\frac{\mathit{\Delta}\overline{H}_t}{H_{t-1,t}}\right] \cdot \mathbb{E}[r_s]\right) \\
&\quad - \frac{\gamma}{2}\omega_H \left(\mathbb{E}\left[\mathrm{Var}_{t-1}(\epsilon_t) \cdot r_s\right] - \mathbb{E}\left[\mathrm{Var}_{t-1}(\epsilon_t)\right] \cdot \mathbb{E}[r_s]\right) \\
&\quad + \frac{\gamma(\gamma+1)}{6}\omega_H^2 \left(\mathbb{E}\left[\mathit{\Delta}\mathrm{Skew}_{t-1}(\epsilon_t) \cdot r_s\right] - \mathbb{E}\left[\mathit{\Delta}\mathrm{Skew}_{t-1}(\epsilon_t)\right] \cdot \mathbb{E}[r_s]\right).
\end{aligned}
$$

This can be expressed as

$$
\begin{aligned}
\mathrm{Cov}\left(\frac{\mathit{\Delta} H_{t-1,t}}{H_{t-1,t}}, r_s\right) &= \mathrm{Cov}\left(\frac{\mathit{\Delta}\overline{H}_t}{H_{t-1,t}}, r_s\right) - \frac{\gamma}{2}\omega_H \cdot \mathrm{Cov}\left(\mathrm{Var}_{t-1}(\epsilon_t), r_s\right) \\
&+ \frac{\gamma(\gamma+1)}{6}\omega_H^2 \cdot \mathrm{Cov}\left(\mathit{\Delta}\mathrm{Skew}_{t-1}(\epsilon_t), r_s\right). \quad \text{(IA13)}
\end{aligned}
$$

Dividing both sides by σ_s^2 to get the expression for β_H, we have

$$
\begin{aligned}
\beta_H &= \frac{\mathrm{Cov}\left(\frac{\mathit{\Delta}\overline{H}_t}{H_{t-1,t}}, r_s\right)}{\sigma_s^2} - \frac{\gamma}{2} \cdot \omega_H \frac{\mathrm{Cov}\left(\mathit{\Delta}\mathrm{Var}(\epsilon), r_s\right)}{\sigma_s^2} + \frac{\gamma(\gamma+1)}{6} \cdot {\omega_H}^2 \frac{\mathrm{Cov}\left(\mathit{\Delta}\mathrm{Skew}(\epsilon), r_s\right)}{\sigma_s^2} \\
&\approx \frac{\mathrm{Cov}\left(\frac{\mathit{\Delta} H_{t-1,t}}{H_{t-1,t}}, r_s\right)}{\sigma_s^2}. \quad \text{(IA14)}
\end{aligned}
$$